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L_∞ identification and model reduction for robust conbtrol

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Abstract: *This paper develops a Boltzmann learning enhanced genetic algorithm for infinity-norm based system identification and model reduction for robust control applications. Using this technique, both a globally optimised nominal model and an error bounding function for additive and multiplicative uncertainties can be obtained. It can also offer a tighter L_∞ error bound and is applicable to both continuous and discrete-time systems.*

1. INTRODUCTION

With rapid development in robust control theory and algorithms, system identification techniques which are compatible with the frequency domain mini-max design framework have received increasing attention. Such techniques should identify a nominal model that best matches the given experimental data in terms of the infinity-norm and provides an uncertainty bound, or an upper limit, of the identification error.

Recently, many useful methods for robust control oriented system identification have been developed in the frequency domain (Helmicki *et al* 1991, 1993; Gu and Khargonekar 1992). However, these methods have failed to provide information on the uncertainty bounds and the resulting nominal models tend to be of high order in general. A few time domain set-membership (Kosut *et al* 1992) and interpolation (Zhou and Kimura 1994) approaches manage to provide relatively low orders, but an accurate uncertainty bounding function must be given *a priori*, which is hardly feasible in practice.

The set-membership based L_∞ reduction approach proposed by Sugie and Tanai (1995) succeeds in identifying both a nominal model and a uncertainty bounding function. However, it requires the nominal model and the bounding function to be linear in parameterisation.

Other sub-optimal approaches include the Balanced truncation method (Chiang and Safonov 1992) and the Hankel norm approximation techniques (Glover 1984). The frequency-weighted H_∞ reduction technique introduced by Zhou (1995) offers good H_∞ norm error bounds if the weighting function is completely unstable and its zeros are all in the right half s-plane.

In this paper, a Boltzmann learning enhanced genetic algorithm (GA) based method is developed to solve infinity-norm based identification and model reduction problems. This evolutionary technique combines the global search power from the “generational” Darwinianism with the interactive fine-learning capacity from the “biological” Lamarckism, overcoming the well-known problems of chromosome stagnation and weak local exploration of genetic algorithms. The identification tasks are highlighted in Section 2. The algorithm is detailed in Section 3, with illustrated examples showing high performances of the proposed method. Finally, conclusions are summarised in Section 4.

2. PROBLEM FORMULATION

2.1 L_∞ System Identification

This section highlights the problems in system identification for robust control in the frequency domain. In the context of identification for robust control, the infinity-norm is adopted as the cost function for modelling (Helmicki *et al* 1991).

2.1.1 Additive Uncertainty Model

Suppose the plant $G(s)$ is to be identified using a nominal model $G_r(s)$ with an additive uncertainty as described by (Doyle and Stein 1981; Sugie and Tanai 1995):

$$G(s) = G_r(s) + \Delta(s)W(s) \tag{1}$$

where $\Delta(s)$ is unknown but is bounded as given by

$$\|\Delta(s)\|_\infty \le 1 \tag{2}$$

Define

$$H(s) = G(s) - G_r(s) \tag{3}$$

to represent the error transfer function resulting from the approximate model.

The infinity-norm identification objective is to find an optimal $G_r(s)$ such that the cost function

$$\begin{aligned} J_G &= \|W_a(s)H(s)\|_\infty = \|W_a(s)\Delta(s)W(s)\|_\infty \\ &\le \|W_a(s)W(s)\|_\infty \end{aligned} \tag{4}$$

is minimised given a frequency weighting function $W_a(s)$. Although J_G is always bounded by $\|W_a(s)W(s)\|_\infty$, the tighter the bound, the more accurate the nominal model can reflect the actual plant dynamics and, thus, the higher the potential performance of the robust controller may offer. Therefore, in the identification, another objective is to find an optimal $W(s)$ such that the cost

$$J_W = \| \, W_b(s)[W(s) - H(s)] \, \|_\infty \tag{5}$$

is minimised under the constraint (Sugie and Tanai 1995):

$$\|H(s)W^{-1}(s)\|_\infty \le 1 \tag{6}$$

derived from (2) and (3).

Here, $W_b(s)$ is also a frequency weighting function. The introduction of such a scaling function allows fitting errors in chosen parts of the frequency range to have a particular emphasis if needed. Since the degree of the uncertainty of the nominal model error depends on the frequency, a meaningful uncertainty bound must be frequency weighted and it is just such frquency weight that reflects the controller design performance.

Note that, it is impossible to minimise both J_G and J_W for an infinite number of frequency points, as the true plant is unknown and the identification is carried out from the plant frequency response data. However, minimising the approximate cost

$$J_G = \max_{k \in \{1,\dots,n\}} |W_a(jw_k)H(jw_k)| \tag{7}$$

over the interested frequency range is possible and is adopted in practice (Zhou and Kimura 1993). Similarly, based on the obtained nominal model, the uncertainty bounding function is to be determined by minimising

$$J_W = \max_{k \in \{1,\dots,n\}} \left| \, W_b(jw_k)[W(jw_k) - H(jw_k)] \, \right| \tag{8}$$

subject to the constraint:

$$\max_{k \in \{1,\dots,n\}} |H(jw_k)W^{-1}(jw_k)| \le 1 \tag{9}$$

where n is a finite number of points covering the frequency range concerned.

2.1.2 Multiplicative Uncertainty Model

A multiplicative uncertainty model of a continuous-time system is represented by (Doyle *et al* 1992):

$$G(s) = G_r(s)[1 + \Delta(s)W(s)] \tag{10}$$

where $W(s)$ is now the multiplicative uncertainty bounding function. Again, the same infinity-norm cost of J_G of (7) is used to identify an optimal nominal model $G_r(s)$. In this case, however, the corresponding multiplicative uncertainty bounding function $W(s)$ can be obtained by minimising the cost (Doyle *et al* 1992):

$$J_W = \max_{k \in \{1,\ldots,n\}} \left| W_b(jw_k) \left\{ W(jw_k) - \left[\frac{G(jw_k)}{G_r(jw_k)} - 1 \right] \right\} \right| \quad (11)$$

subject to the constraint:

$$\max_{k \in \{1,\ldots,n\}} \left| \left[\frac{G(jw_k)}{G_r(jw_k)} - 1 \right] W^{-1}(jw_k) \right| \le 1 \quad (12)$$

2.1.3 Discrete-Time Systems

In the discrete-time, the additive and multiplicative uncertainty model descriptions are:

$$G(z) = G_r(z) + \Delta(z)W(z) \quad (13)$$

and

$$G(z) = G_r(z)[1 + \Delta(z)W(z)] \quad (14)$$

respectively. Again,

$$\|\Delta(z)\|_\infty \le 1 \quad (15)$$

All derivations in Subsections 2.1 and 2.2 hold by replacing jw with e^{jwT} and by changing the argument s to z. Note that, however, the infinity-norm for a discrete-time system is given by:

$$\|G(z)\|_\infty = \sup_{|wT| \le p} \left| G(e^{jwT}) \right| \quad (16)$$

where T is the sampling period.

2.2 Frequency-weighted L_∞ Norm Model Reduction

Given a transfer function $G(s)$, the objective of frequency-weighted infinity-norm model reduction is to find a transfer function $G_r(s)$ such that the cost $\| W_a(s)[G(s) - G_r(s)] \|_\infty$ in (4) is minimised. The following lemma provides a lower bound for an infinity-norm approximation (Glover 1984).

Lemma 1 *Given an m^{th} order transfer function $G(s)$, there is an r^{th} order $G_r(s)$ such that $s_{r+1} \le \|G - G_r\|_\infty$, where s_{r+1} is the $(r+1)^{th}$ Hankel Singular Values of G.*

Unlike Hankel norm model reduction, the lower bound is not necessarily achievable in the infinity-norm model reduction problem.

Computations for an optimal solution to the above identification problems are in general an open issue. These problems are difficult to solve using conventional optimisation techniques, such as the least mean-squares method (Helmicki *et al* 1993), due to the complexity and multimodality of the problems. Also, these conventional methods require a differentiable error energy function and appropriate initial conditions. A powerful evolutionary optimisation based method is thus to be developed here to solve the infinity-norm based identification problems.

3. L_∞ IDENTIFICATION AND MODEL REDUCTION BY EVOLUTION

3.1 Boltzmann Learning Enhanced Genetic Algorithm

The *genetic algorithm* (Goldberg 1989) acts on the "survival-of-the-fittest" Darwinian-Wallace principle that emulates natural selection and genetics. A GA searches the solution space in parallel, evaluating a population of potential solutions. It is particularly effective in approaching the global optimum in a noisy, poorly understood and/or non-differentiable search space. A basic GA involves three types of operations: *reproduction, crossover and mutation*. (see Goldberg 1989 for details of the algorithm).

The search power of a GA mainly lies in its crossover and mutation operators, which provide the diversity of candidate “species” (i.e., identified models) and varying search domains. However, it is well-known that existing GAs are weak in local exploration and are thus poor in finding the exact optima at each generation (Tan *et al.*, 1995). The underlying reason of this is that, in a pure GA, there is a lack of “biological diversity” resulting from interactions with, and thus direct learning from, the evolution environment, termed the "Lamarckism inheritance" (Aboitiz, 1992).

To improve the GA performance, this paper attempts to combine the "generational" optimisation power of crossover of an evolving population with the Lamarckism "inheritance" of learning individuals in each generation. "Positive mutations" resulting from learning can again be implemented through trial-and-error, requiring no derivative information. Here a Boltzmann type of learning is realised by simulated annealing (SA) (Tan *et al.*, 1996), which asserts a probability of retaining possibly correct search directions. As shown in Fig 1, an existing chromosome C_k (a potential parameter set) may be replaced by its mutant chromosome C with a probability given by the Boltzmann selection criterion:

$$P\left(C \leftarrow C_k \middle| C_k\right) = \min\left\{\exp\left[-\frac{J(C) - J(C_k)}{k_{\mathrm{B}}T}\right], 1\right\} \tag{17}$$

where C may be slightly inferior. Here k_{B} may be the Boltzmann's constant, but is in this paper set to 5×10^{-5} in the artificial annealing process. The "annealing temperature" T decreases from T_{ini} exponentially at the rate of $b^{\,j-1}$, where $b < 1$ is the annealing factor and the integer $j \in [1, j_{\max}]$ is the annealing cycle index. The decreasing temperature in Fig 1 implies that the learning mechanism will move close to hillclimbing. The final temperature T_{final} is determined by how tight the fine-tuning should be bounded at the end of the learning process. Here, a fast annealing scheme (b = 30%) is used, because the major task of optimisation is undertaken by the global evolution. For a final learning tolerance given by $T_{\mathrm{final}} = 1$ and an initial tolerance by $T_{\mathrm{ini}} = 10^5$, the total number of annealing trials is:

$$j_{\max} = \left\lceil \log_b \frac{T_{\mathrm{final}}}{T_{\mathrm{ini}}} \right\rceil = 10 \tag{18}$$

In this paper, the chromosome length and *Hamming cliff* effect encountered in the GA binary coding are reduced by decimal coding (Li, 1995). A range coding scheme (Ng, 1995) is also used, where each parameter is coded by 4 digits, the first of which is used to evolve and learn an appropriate range of parameter values. The rest 3 digits represent a quantified candidate value within the selected range. In Fig 1, the *tournament selection* scheme is employed for rapid reproduction, where two random chromosomes compete once for survival. Half of the winning 50% chromosomes resulting from the tournament will be further trained for Lamarkian heredity. The mature spring will be mixed with the winning individuals and 25% of the parents to form a new population. This population formation mechanism is similar to a "steady-state GA" (Goldberg, 1989), which reserves some possibly useful genes and creates a *generation gap* of 0.25. It helps to save the function evaluation time and tends to maintain a diversity of the species for global optima.

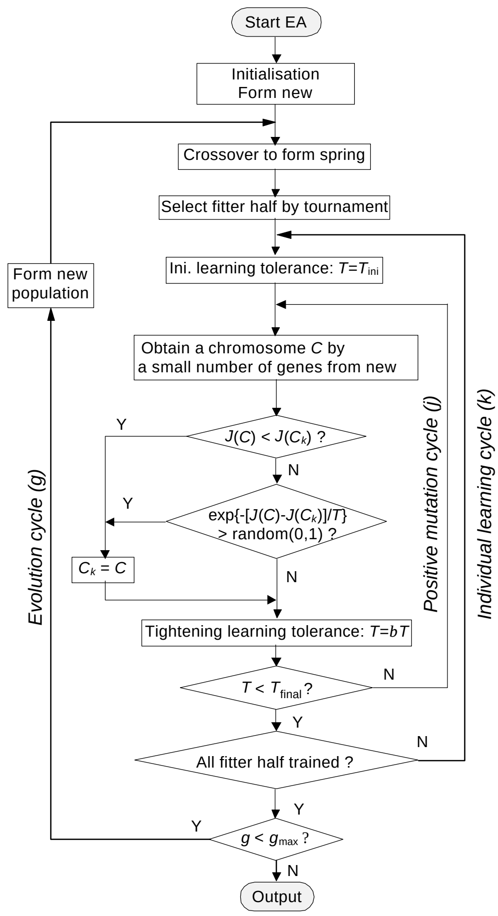

Fig. 1. Flow chart of a GA using "positive mutations" with a Boltzmann learning schedule.

3.2 L_∞ System Identification Example

Assume a 4th order discrete plant of an industrial heat exchanger (Golten and Verwer 1991) given by:

$$G(z)=\frac{0.049(z+0.72)}{z^2(z-0.607)^2} \tag{19}$$

with a sampling period of T = 7.5 s is to be modelled by a discrete-time additive uncertainty model with a first order uncertainty bounding function. The discrete-time frequency weighting functions in this case are chosen as $W_a(z)$ = 1 and

$$W_b(z)=\frac{7.5z-4.3}{z-0.34} \tag{20}$$

The GA identified second and third order discrete nominal models are given by:

$$G_{r,2nd}(z)=\frac{-0.0225z+0.0628}{z^2-1.5856z+0.668} \tag{21}$$

$$G_{r,3rd}(z)=\frac{-0.00027z^2-0.004z+0.067}{z^3-1.3954z^2+0.5155z-0.00335} \tag{22}$$

and their uncertainty bounding functions are:

$$W_{2nd}(z)=\frac{0.0635}{z} \tag{23}$$

$$W_{3rd}(z)=\frac{0.01923}{z} \tag{24}$$

Frequency responses of the true discrete plant and the discrete nominal models are plotted in Fig. 2. It can be seen that the proposed hybrid evolutionary technique gave excellent identification results, with a good fitting over the frequency range concerned.

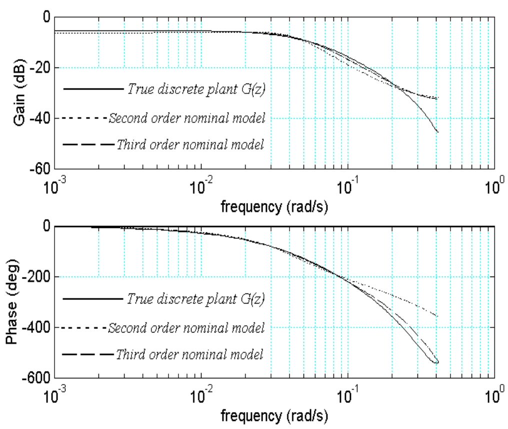

Fig. 2. Frequency responses of the true discrete plant and the discrete nominal models

3.3 L_∞ Model Reduction Example

Study the 4th order transfer function given by Anderson (1986) and Zhou (1995):

$$G(s)=\frac{s^2+0.2s+1.01}{s^2+0.2s+4.04}\,\frac{s^2+0.2s+9.01}{s^2+0.2s+16.02} \tag{25}$$

with a frequency weighting function,

$$W_a(s)=\frac{(s-1)^2}{s^2-0.2s+1} \tag{26}$$

The optimally identified second and third order reduced models from the GA are

$$G_{r,2nd}(s)=\frac{0.96s^2+2.7448s+2.31}{s^2+0.459s+17.146} \tag{27}$$

$$G_{r,3rd}(s)=\frac{4.2526s^3+8.033s^2+81.802s+1.216}{s^3+12.2327s^2+19.4596s+200.3} \tag{28}$$

Table 1 gives the model reduction errors of the GA based method and the various well known methods in the literature. It can be seen that the evolutionary and learning technique outperforms others, yielding the smallest L_∞ norm errors for both the 2nd and 3rd order reductions.

Table 1. The infinity-norm errors in $\left\| W_a(s)[G(s) - G_r(s)] \right\|_\infty$

Identified Model Order	2nd	3rd
Lower Bounds (HSV)	2.704	2.527
Latham & Anderson (1986)	20.08	11.94
Chiang & Safonov (1992)	11.71	6.303
Zhou (1995) (Algorithm I)	4.827	8.20
Zhou (1995) (Algorithm II)	4.822	3.946
Learning GA	4.517	3.789

In table 1, *HSV* stands for the $(r+1)^{th}$ Hankel singular value of $W_\alpha(s)G(s)$, which is the lower bound for the optimal error by Lemma 1. The frequency-weighted errors for both the second and the third order reduced models are shown in Fig. 3. Responses shown that the hybrid GA gave a small and tight infinity-norm error bounds for both the 2nd and 3rd order reductions.

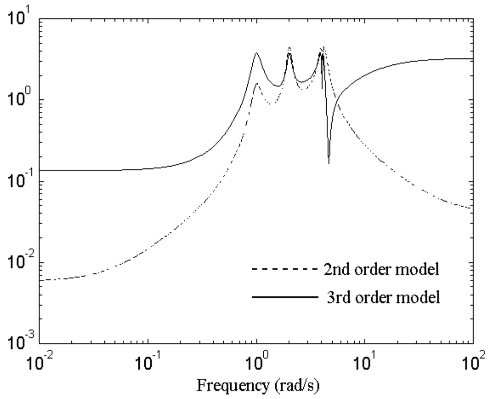

Fig. 3. Gain plots of the weighted model reduction errors

4. CONCLUSION

This paper has developed a Boltzmann learning enhanced genetic algorithm technique for system identification and model reduction in the context of robust control. It is shown that both an optimal nominal model and an uncertainty bounding function can be obtained by a GA globally minimising the infinity-norm costs. The technique also provides a tighter infinity-norm error bound than existing methods for the model reduction problem. The hybrid evolutionary approach is applicable to both continuous and discrete-time systems. Examples show that this method provides a uniform tool and high accuracy for "worst case" identification and reduction. It is currently applied to MIMO systems. Results will be reported in due course.

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